

Пример решения задачи 19 типового расчета

$$\lim_{x \rightarrow 0} \frac{\sin 3x + \ln(1-2x) - \operatorname{tg} x}{\sqrt[3]{1+3x} - \sqrt{1+2x}}$$

$$\sin 3x = 3x + \bar{o}(x^2)$$

$$\ln(1-2x) = -2x - \frac{(-2x)^2}{2} + \bar{o}(x^2) = -2x - 2x^2 + \bar{o}(x^2)$$

$$\operatorname{tg} x = x + \bar{o}(x^2)$$

$$\text{числитель} = 3x - 2x - x - 2x^2 + \bar{o}(x^2) = -2x^2 + \bar{o}(x^2)$$

$$\sqrt[3]{1+3x} = 1 + \frac{1}{3} \cdot 3x + \frac{\frac{1}{3} \left(\frac{1}{3} - 1 \right)}{2!} \cdot (3x)^2 + \bar{o}(x^2) = 1 + x - x^2 + \bar{o}(x^2)$$

$$\sqrt{1+2x} = 1 + \frac{1}{2} \cdot 2x + \frac{\frac{1}{2} \left(\frac{1}{2} - 1 \right)}{2!} \cdot (2x)^2 + \bar{o}(x^2) = 1 + x - \frac{1}{2}x^2 + \bar{o}(x^2) = -\frac{1}{2}x^2 + \bar{o}(x^2)$$

$$\text{знаменатель} = 1 + x - x^2 - 1 - x + \frac{1}{2}x^2 + \bar{o}(x^2) = -\frac{1}{2}x^2 + \bar{o}(x^2)$$

$$\lim_{x \rightarrow 0} \frac{\sin 3x + \ln(1-2x) - \operatorname{tg} x}{\sqrt[3]{1+3x} - \sqrt{1+2x}} = \lim_{x \rightarrow 0} \frac{-2x^2 + \bar{o}(x^2)}{-\frac{1}{2}x^2 + \bar{o}(x^2)} = \lim_{x \rightarrow 0} \frac{-2 + \bar{o}(1)}{-\frac{1}{2} + \bar{o}(1)} = 4$$